

**NON-LINEAR SYSTEM AND SUBSYSTEM MODELLING IN THE TIME DOMAIN**

M.I. Sobhy, E.A. Hosny, M.W.R. Ng and El-Sayed A. Bakkar

The Electronic Engineering Laboratories,  
 The University of Kent at Canterbury  
 Canterbury, Kent, CT2 7NT, U.K.

**ABSTRACT**

This work describes how non-linear subsystems can be modelled from time domain measurements. The advantage being the simplicity of the measurement and the developed models and the speed of the simulation of the entire system.

**TIME DOMAIN MODELLING**

Linear and non-linear subsystems can be modelled using time domain measurements and then the entire system consisting of the individual subsystems can then be very effectively analysed [1,2]. The models of the subsystem do not use conventional equivalent circuit but consist of the following elements.

- . Adders
- . Non-linear multipliers
- . Linear multipliers
- . Differentiators
- . Integrators
- . Linear and non-linear delays

This approach gives simpler and more accurate models than the equivalent circuit approach as it dispenses with the need to obey Kirchhoff's laws. One problem arises from this approach is the stability of the developed models. Stability can be guaranteed if no feed back is used however this restriction may increase the complexity of the model. The advantages of this approach are summarized below:

- . A large system can be divided into smaller subsystems and the models for each subnetwork are derived separately.
- . A library of subsystems models can be

developed and stored for future use without the need of an equivalent circuit. This includes amplifiers, mixers, oscillators, filters and couplers.

The model characterising a non-linear subsystem can be derived to match experimental data without the need to develop a physically realizable equivalent circuit. This gives a greater flexibility in modelling active devices.

With this approach each subsystem is identified separately and a time domain model for each scattering parameter is developed. The effect of the power supply can also be identified. For example a non-linear amplifier can be regarded as a three-port network with the power supply input as the third port in the network.

**TIME DOMAIN S-PARAMETERS  
MEASUREMENT SYSTEM**

The incident, reflected and transmitted waves which are used to identify the S-parameters of the subsystems are measured in time domain. A set up for measuring an amplifier is shown in Fig. 1. Incident pulse wave from the generator

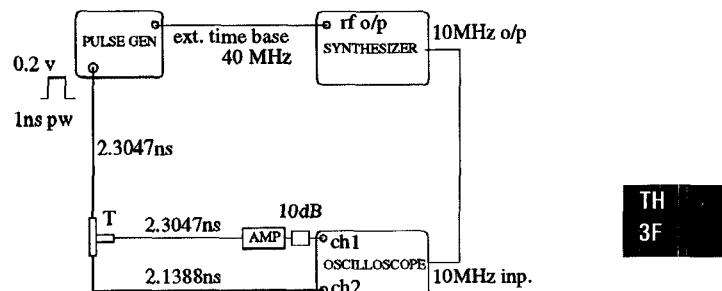


Fig. 1 Time domain S-parameters measurement system for amplifier

travels towards the tee connector and splits into two equal parts. One travels through the amplifier and enters channel 1 of the sampling oscilloscope as the transmitted wave. The other part travels towards the channel 2 as the incident wave. The reflected wave from the amplifier travels back to the tee connector. One part returns to the generator. Another part travels to the channel 2 as the reflected wave. In Channel 2, the incident and reflected waves are separated in time by twice the delay of the transmission line between the tee connector and the amplifier. The measured waveforms are shown in Fig. 2 and 3. The system is calibrated using a short circuit and a matched termination to remove the errors introduced by the tee connector and the transmission lines

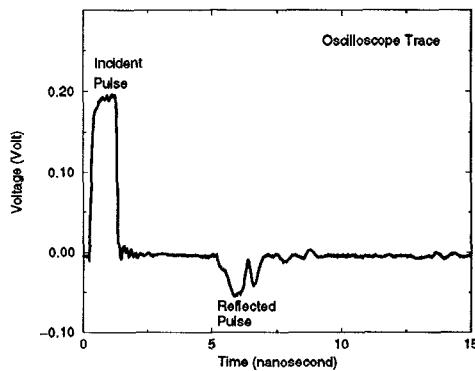


Fig. 2 Measured incident and reflected waveforms of amplifier

The choice of incident waveform is important to identify accurate model of the subsystem in time domain. Pulse wave is found to be optimal for amplifiers and filters in terms of accuracy and efficiency. The pulse parameters should be chosen such that the usable spectrum cover the operating input frequency and amplitude of the subsystem to be identified. Sinusoidal incident wave is found to be most suitable to identify model of mixer due to the complicate output waveform from mixing. The incident, reflected and transmitted waves are also measured in time domain. Directional couplers are used to separate the incident and the reflected waves.

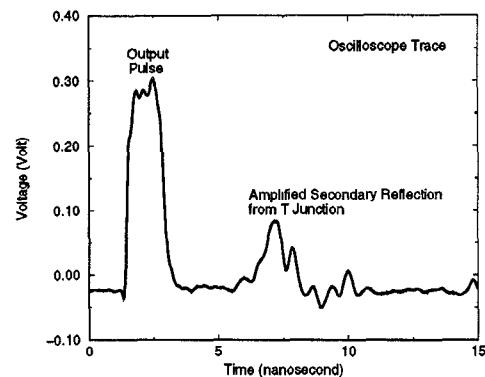


Fig. 3 Measured transmitted waveforms of amplifier

## MODELS OF SUBSYSTEMS

A Mini-Circuits ZFL-1000LN Amplifier has been modelled in time domain [3]. The four S-parameters are non-linear time domain transfer functions as shown in Fig. 4.

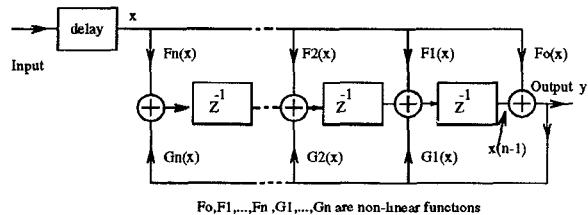


Fig. 4 Non-linear time domain transfer function diagram

It is an auto-regressive moving average (ARMA) filter in non-linear form. A Mini-Circuits SIF-70 Band Pass Filter and SLP-1200 low pass filter have also been modelled in time domain [3]. The four S-Parameters are linear time domain transfer functions. A Mini-Circuit ZEM-4300 double balance mixer has also been modelled in

time domain [3]. The main mixing time domain non-linear transfer function as shown in Fig.5 consists of a multiplier to simulate the mixing effect. Non-linear function and filter are used to provide the wide band modelling capability.

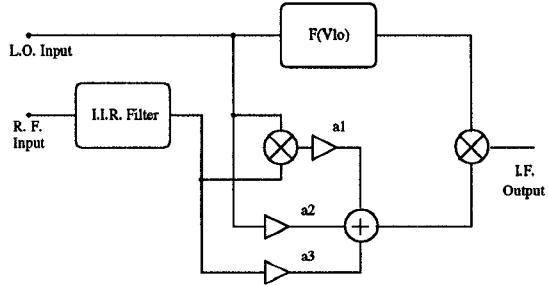


Fig. 5 Structure of mixer model

A Z-comm L-351 voltage controlled oscillator is modelled in time domain. The structure is shown in Fig. 6.

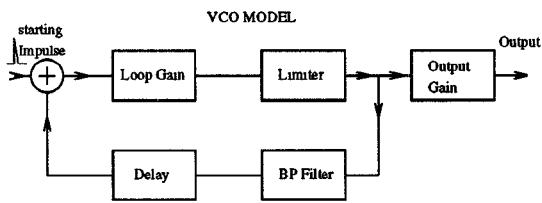


Fig. 6 Structure of voltage controlled oscillator model

The oscillation is started by an impulse and maintained by positive feedback. The loop gain, delay and output gain are functions of the tuning voltage. The output frequency of the VCO model and measurements are shown in Fig. 7. The reflection coefficient of the VCO is identified in form of non-linear time domain transfer function as shown in Fig. 4.

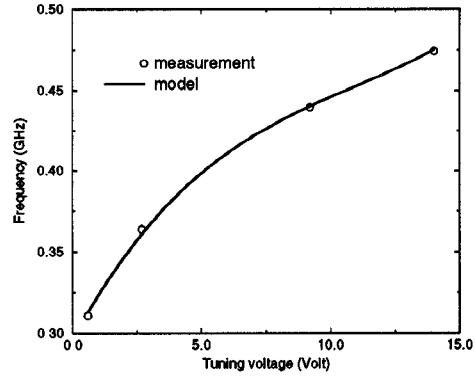


Fig. 7 Tuning characteristic of the voltage controlled oscillator

### COMPLETE SYSTEM SIMULATION

From the results of time domain measurement, each subsystem is represented by their scattering parameters in the time domain. The entire system can then be built up of the individual subsystems in any desired connection. Fig. 8. shows how three subsystems, a low pass filter, an amplifier and a band pass filter for example can be cascaded.

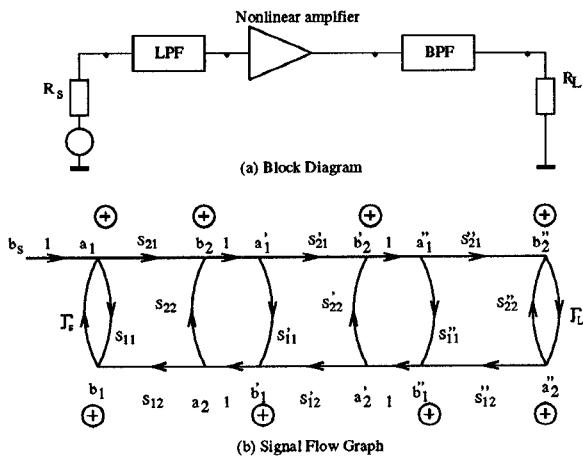


Fig. 8 S-parameters signal flow diagram of a cascaded non-linear system

The various signals in the flow chart are related by

$$\begin{aligned}
 a_1 &= b_s + b_1 \Gamma_s, \\
 b_1 &= a_1 S_{11} + a_2 S_{12}, \\
 b_2 &= a_1 S_{21} + a_2 S_{22}, \\
 a'_1 &= b_2, \\
 a_2 &= b'_1, \\
 b'_1 &= a'_1 S'_{11} + a'_2 S'_{12}, \\
 b'_2 &= a'_1 S'_{21} + a'_2 S'_{22}, \\
 a''_1 &= b'_2, \\
 a'_2 &= b''_1, \\
 b''_1 &= a''_1 S''_{11} + a''_2 S''_{12}, \\
 b''_2 &= a''_1 S''_{21} + a''_2 S''_{22}, \\
 a''_2 &= b''_2 \Gamma_L
 \end{aligned}$$

At each time step, the above equations are solved to obtain the incident and reflected parameters. In all microwave networks, the S-parameters will have a certain delay and hence every branch in the signal flow chart contains a delay. The approach is similar to wave digital filter representations without any delay-free loop. In this case the solutions of the above equations at time  $t$  only depend on the previous values of the signals and no iterations are required as in the case of the equivalent circuit approach. This is true even if the circuit is non-linear. A system consisting of a low pass filter, a non-linear amplifier and a band pass filter was simulated.

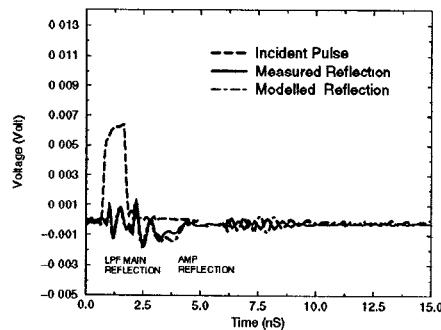


Fig. 9 Reflected waveform of the cascaded non-linear system

Every individual subsystem was measured in the time domain and a model for each of the four S-parameters was developed. Then the entire system was modelled and the results compare to measurements. Fig. 9 shows the time domain reflection results and Fig. 10 shows the effect of increasing the input signal amplitude to the transmission results.

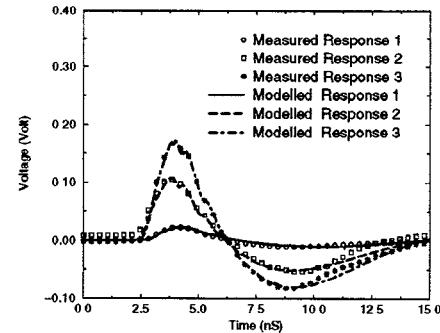


Fig. 10 Transmitted waveforms of the cascaded non-linear system

## CONCLUSIONS

A new method of subsystem modelling and system simulation has been developed. Non-linear modelling and simulation are performed in the time domain. Black box model is used instead of equivalent circuit model. The system simulation can deal with any number of subsystems and any arbitrary connection. A cascade non-linear system is modelled and the simulation results agree with measurements.

## REFERENCES

- [1] M.I. Sobhy, E.A. Hosny and M.A. Nassef "Multiport Approach for the Analysis of Microwave Non-linear Networks". International Journal of Numerical Modelling. Vol 28, No. 1 Jan. 1993
- [2] E.A. Hosny, M.A. Nassef and M.I. Sobhy "Computer-aided Analysis of Non-linear Lumped and Distributed Multiport Networks". Proc. IEEE International Symposium on Circuits and Systems. Vol. 1 pp. 418-431, New Orleans, May 1990
- [3] M.I. Sobhy, E.A. Hosny, M.W.R. Ng and El-Sayed A. Bakkar "Time Domain Measurement and Identification of Microwave Systems". pp.641-645, Proc. 25th European Microwave Conference, Bologna, Sept. 1995